4.4) $A_1 = \begin{bmatrix} -3 & 1 & -3 \\ 20 & 3 & 10 \\ 2 & -2 & 4 \end{bmatrix}$

$P(\lambda) = \det(\lambda I - A_1) = \det\left(\begin{bmatrix} \lambda+3 & -1 & 3 \\ -20 & \lambda-3 & -10 \\ -2 & 2 & \lambda-4 \end{bmatrix}\right) =$

$= (\lambda+3).[(\lambda^2-7\lambda+12)+20] + (-20\lambda+80-20) + 3.[-40-(-2\lambda+6)] =$

$= (\lambda+3).(\lambda^2-7\lambda+32) - 20\lambda+60-138+6\lambda =$

$= \lambda^3 - 7\lambda^2 + 32\lambda + 3\lambda^2 - 21\lambda + 96 - 14\lambda - 78 =$

$= \lambda^3 - 4\lambda^2 - 3\lambda + 18$

Autoval. $\rightarrow \lambda^3 - 4\lambda^2 - 3\lambda + 18 = 0$ $\rightarrow$ $\boxed{\lambda_1 = -2,\ \lambda_2 = 3,\ \lambda_3 = 3}$

Para $\lambda = -2$

$\begin{pmatrix} 1 & -1 & 3 \\ -20 & -5 & -10 \\ -2 & 2 & -6 \end{pmatrix} \begin{matrix} \\ F2 \to 20F1+F2 \\ F3 \to 2F1+F3 \end{matrix} \begin{pmatrix} 1 & -1 & 3 \\ 0 & -25 & 50 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{cases} x - y + 3z = 0 \rightarrow x = -z \\ -25y + 50z = 0 \rightarrow y = 2z \end{cases}$

$\rightarrow \bar{x} = (-z; 2z; z) = z.(-1; 2; 1)$

$\boxed{\text{AUTOVECTOR } \lambda = -2: (-1; 2; 1)}$

Para $\lambda = 3$

$$\begin{pmatrix} 6 & -1 & 3 \\ -20 & 0 & -10 \\ -2 & 2 & -1 \end{pmatrix} \begin{matrix} F2 \to 20F1 + 6F2 \\ F3 \to F1 + 3F3 \end{matrix} \begin{pmatrix} 6 & -1 & 3 \\ 0 & -20 & 0 \\ 0 & 5 & 0 \end{pmatrix} F3 \to 5F2 + 20F3$$

$$\begin{pmatrix} 6 & -1 & 3 \\ 0 & -20 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{matrix} 6x - y + 3z = 0 \to 6x = -3z \to z = -2x \\ -20y = 0 \to y = 0 \end{matrix}$$

$$\to \bar{x} = (x, 0, -2x) = x \cdot (1, 0, -2)$$

Multiplicidad geometrica de $\lambda = 3$ (1) menor a su multiplicidad algebraica (2). No es diagonalizable A_1.

$$A_2 = \begin{bmatrix} -4 & -3 & -3 \\ 0 & -1 & 0 \\ 6 & 6 & 5 \end{bmatrix}$$

$$P(\lambda) = \det(\lambda I - A_2) = \det\left(\begin{bmatrix} \lambda+4 & 3 & 3 \\ 0 & \lambda+1 & 0 \\ -6 & -6 & \lambda-5 \end{bmatrix}\right)$$

$$= (-\lambda - 1) \cdot [(\lambda^2 - \lambda - 20) + 18] = (-\lambda - 1) \cdot (\lambda^2 - \lambda - 2) =$$

$$= -\lambda^3 \cancel{+ \lambda^2} + 2\lambda \cancel{+ \lambda^2} + \lambda + 2 = -\lambda^3 + 3\lambda + 2$$

Autoval. $\to \lambda^3 - 3\lambda - 2 = 0$

$$\boxed{\begin{matrix} \to \lambda = -1 \\ \to \lambda_2 = -1 \\ \to \lambda_3 = 2 \end{matrix}}$$

Para $\lambda = -1$

$$\begin{pmatrix} 3 & 3 & 3 \\ 0 & 0 & 0 \\ -6 & -6 & -6 \end{pmatrix} \xrightarrow{F3 \to 2F1+F3} \begin{pmatrix} 3 & 3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to 3x+3y+3z=0$$

$$\to x = -3y-3z$$

$$\to \bar{x} = (-3y-3z;\ y;\ z) = y.(-3;1;0) + z.(-3;0;1)$$

$$\boxed{\text{AUTOVECT. } \lambda=-1: \{(-3;1;0),\ (-3;0;1)\}}$$

Para $\lambda = 2$

$$\begin{pmatrix} 6 & 3 & 3 \\ 0 & 3 & 0 \\ -6 & -6 & -3 \end{pmatrix} \xrightarrow{F3 \to F1+F3} \begin{pmatrix} 6 & 3 & 3 \\ 0 & 3 & 0 \\ 0 & -3 & 0 \end{pmatrix} \xrightarrow{F3 \to F2+F3} \begin{pmatrix} 6 & 3 & 3 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\to \begin{cases} 6x+3y+3z=0 \to 6x=-3z \to z=-2x \\ 3y=0 \to y=0 \end{cases}$$

$$\to \bar{x} = (x, 0, -2x) = x.(1,0,-2)$$

$$\boxed{\text{AUTOVECT. } \lambda=2: \{(1,0,-2)\}}$$ DIAGONALIZABLE.

$$\Lambda = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad P = \begin{bmatrix} -3 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & -2 \end{bmatrix}$$